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FUZZY GAME PROBLEM WITH PAYOFFS AS LINGUISTIC VARIABLES

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ABSTRACT
In this paper, we investigate a new method to solve a fuzzy game problem with payoff as represented by linguistic variables which are replaced by Heptagonal fuzzy numbers is developed for each of the players. By using ranking Procedure on these Heptagonal fuzzy numbers and convert the fuzzy game problem into crisp game problem and solved by pivot method. Finally, a numerical example is given to illustrate the solution procedure.

INTRODUCTION
The history of game theory dates back to the early twentieth century but a new turn for its wider applicability took only in 1944, when John von Newmann and Oscar Morgenstern published the famous article 'Theory of games and economic Behavior' [12].A game is a decision making situation with many players, having objectives that partly or completely conflict with each other. Using the notion of fuzzy sets, each component in a game (set of players, set of strategies, set of payoffs,etc) can be fuzzified. Fuzzy set theory was formally introduced by Lotfi. A.Zadeh in his classic paper " Fuzzy sets" in the year 1965 [18]. Fuzzy numbers play an important role in many applications, including decision making, approximate reasoning, optimization etc. Fuzzy numbers are widely used in the research on fuzzy set theory. In fuzzy game problems, all parameters are fuzzy numbers. Fuzzy numbers may be normal or abnormal, triangular or trapezoidal and these fuzzy numbers are not directly comparable. Several methods are introduced for ranking of fuzzy numbers. Ranking fuzzy numbers are an important tool in decision making. Various ranking procedure have been developed since 1976, when the theory of fuzzy sets were first introduced by Zadeh [18]. We investigate a fuzzy game problem with payoff as represented by linguistic variables which are replaced by Heptagonal fuzzy number. By using ranking to the payoffs, the fuzzy game problem has been transformed into a crisp one, using linguistic variables and solved by pivot method.

PRELIMINARIES
2.1. Fuzzy Set Theory [17,19]
A fuzzy set is characterized by a characteristic function, which assigns to each object a grade of membership ranging between zero and one. The fuzzy set can be shown with $\tilde{A}$ and the membership function $\mu_{\tilde{A}}(x)$. $\mu_{\tilde{A}}(x)$ is a real number between the interval of [0,1]. The membership of each element 'x' is defined by fuzzy grade of this element.

2.2.Fuzzy Number [17]
A fuzzy number $\tilde{A}$ is a fuzzy set on the real line R, must satisfy the following conditions.

(i) There exist atleast one $x_o \in R$ with $\mu_{\tilde{A}}(x_o)=1$
(ii) $\mu_{\tilde{A}}(x)$ is piecewise continuous.
(iii) $\tilde{A}$ must be normal and convex.

HEPTAGONAL FUZZY NUMBER (HFN)
3.1.Definition:[14]
A fuzzy number $\tilde{A}$ is a normal Heptagonal fuzzy number denoted by $(a_1,a_2,a_3,a_4,a_5,a_6,a_7)$ where $a_{1},a_2,a_3,a_4,a_5,a_6,a_7$ are real numbers and its membership function $\mu_{\tilde{A}}(x)$ is given below

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$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & for\ x < a_1 \\ k\left(\frac{x-a_1}{a_2-a_1}\right) & for\ a_1 \le x \le a_2 \\ k & for\ a_2 \le x \le a_3 \\ k+\left(\frac{x-a_3}{a_4-a_3}\right) & for\ \ a_3 \le x \le a_4 \\ k+\left(\frac{a_5-x}{a_5-a_4}\right) & for\ \ a_4 \le x \le a_5 \\ k & for\ \ a_5 \le x \le a_6 \\ k\left(\frac{a_7-x}{a_7-a_6}\right) & for\ \ a_6 \le x \le a_7 \\ 0 & for\ x \ge a_7 \end{cases}$$

3.2 . α- cut of HFN:[14]

If we get crisp interval by α- cut operations interval $A_\alpha$shall be obtained as follows for all

$$\alpha \in [0,1].A_\alpha = \begin{cases} \left[\frac{\alpha}{k}(a_2-a_1)+a_{1,}\ -\frac{\alpha}{k}(a_7-a_6)+a_7\right] & \text{for } \alpha \in [0,k] \\ \left[\left(\frac{\alpha-k}{w-k}\right)(a_4-a_3)+a_3,\ -\left(\frac{\alpha-k}{w-k}\right)(a_5-a_4)+a_5\right] & \text{for } \alpha \in [k,w] \end{cases}$$

3.3.Arithemetic Operations on Heptagonal Fuzzy Number:[4]

Let $\tilde{A}$= (a_1, a_2, a_3,a_4,a_5,a_6, a_7) and $\tilde{B}$= (b_1, b_2, b_3,b_4,b_5,b_6,b_7) are two Heptagonal fuzzy numbers then

1. Addition : $\widetilde{A_H}(+)\widetilde{B_H}$= ($a_1$+ b_1, a_2+ b_2, a_3+ b_3, a_4+ b_4, a_5+ b_5, a_6+ b_6,a_7+b_7)
2. Subtraction : $\widetilde{A_H}(-)\widetilde{B_H}$= ($a_1$- b_7, a_2- b_6, a_3- b_5, a_4- b_4, a_5- b_3, a_6- b_2,a_7-b_1)
3. Scalar Multiplication :

$$\tilde{A} = \begin{cases} (a_{1,}a_{2,}a_{3,}a_{4,}a_{5,}a_{6,}a_7)\ if\ \lambda > 0 \\ (a_{7,}a_{6,}a_{5,}a_{4,}a_{3,}a_{3,}a_1)\ if \lambda < 0 \end{cases}$$

Where λ be a scalar.

3.4. Fuzzy Linguistic Variable [18]

Based on fuzzy set theory introduced by Zadeh, a fuzzy set becomes linguistic variable when it is modified with descriptive words. The concept of a linguistic variable provides a means of approximate characterization of phenomena which are too complex or too ill-defined to be amenable to description in conventional quantitative terms. Linguistic values can be represented using different kinds of fuzzy numbers.

Example: Treating truth as a linguistic variable with value such as true, very true. Completely true, not Very true,etc.

RANKING OF HFN [14]

Let $\tilde{A}_{hep} = (a_{1,}a_{2,}a_{3,}a_{4,}a_{5,}a_{6,}a_7)$ be a heptagonal fuzzy number. A ranking method is from the following formula.

$$\mathrm{R}(\tilde{A}_{hep}) = \frac{a_{1+}a_{2,}+a_3+a_4+a_5+a_6-a_7}{5}$$

For any two Heptagonal fuzzy number,

$$\tilde{A}_{hep} = (a_{1,}a_{2,}a_{3,}a_{4,}a_{5,}a_{6,}a_7) \text{and } \tilde{B}_{hep} = (b_{1,}b_{2,}b_{3,}b_{4,}b_{5,}b_{6,}b_7)$$

We have the following comparison

i. $\tilde{A}_{hep} \approx \tilde{B}_{hep} \Leftrightarrow R(\tilde{A}_{hep}) \approx R(\tilde{B}_{hep})$
ii. $\tilde{A}_{hep} \ge \tilde{B}_{hep} \Leftrightarrow R(\tilde{A}_{hep}) \ge R(\tilde{B}_{hep})$
iii. $\tilde{A}_{hep} \le \tilde{B}_{hep} \Leftrightarrow R(\tilde{A}_{hep}) \le R(\tilde{B}_{hep})$

PROCEDURE FOR DESCRIPTION OF THE PIVOT METHOD FOR SOLVING GAMES

Step (1)

Add a constant to all elements of the game matrix if necessary to insure that the value is positive.[At the end to subtract this constant from the value of the new matrix game to get the value of the original matrix game].

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Step(2) Create a tableau by augmenting the game matrix with a border of -1's along the lower edge , +1's along the right edge, and zero in the lower right corner, Player I's strategy on the left from A_1 to A_n and Player II's strategy on the top from B_1 to B_n.

	B_1	B_2	..	..	B_n	
A_1	a_{11}	a_{12}	..	..	a_{1n}	1
A_2	a_{21}	a_{22}	..	..	a_{2n}	1
…	…	…	..	..	..	..
…	…	…	..	..	..	..
A_n	a_{m1}	a_{m2}	..	..	a_{mn}	1
	-1	-1	..	..	-1	0

Step(3)

Select any entry in the interior of the tableau to be the pivot, say row p and column q, subject to the properties:

(a). The border number in the pivot column, a(m+1,q) must be '-' ve.

(b). The pivot a(p,q), itself must be "+'ve.

(c). The pivot row p, must be chosen to give the smallest of the ratios the border number in the pivot row to the pivot, a(p,n+1)/a(p,q), among all position pivots for that column.

Step(4)

Pivot as follows:

(a) Replace each entry, a(i, j), not in the row (or) column of the pivot by a(i,j)- a(p,j).a(i,q)/a(p,q)

(b) Replace each entry in the pivot row, except for the pivot, by its value divided by the pivot value.

(c) Replace each entry in the pivot column, except for the pivot, by the negative of its value divided by the pivot value.

(d) Replace the pivot value by its reciprocal.

This may be represented by

i.e., $\begin{pmatrix} p & r \\ c & q \end{pmatrix} \to \begin{pmatrix} \frac{1}{p} & \frac{r}{p} \\ \frac{-c}{p} & q-\frac{rc}{p} \end{pmatrix}$

Where 'p' stands for the pivot, 'r' represents any number in the same row as the pivot, 'c' represents any number in the same column as the pivot and q is an arbitrary entry not in the same row(or) column as the pivot

Step (5)

Exchange the label on the left of the pivot row with the label on the top of the pivot column.

Step(6)

If there are any '-'ve numbers remaining in the lower border now, go back to step (3).

Step(7)

Otherwise, a solution may be obtained as

(a) The value, V is the reciprocal of the number in the lower right corner.

(b) The optimal strategy for player I's constructed as follows. Those variables of player I that end up on the left side receive probability zero. Those that end up on the top receive the value of the bottom edge in the same column divided by the lower right corner.

(c) The optimal strategy for player II's constructed as follows. Those variables of player II that end up on the top receive probability zero. Those that end up on the left receive the value of the right edge in the same row divided by the lower right corner.

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NUMERICAL EXAMPLE

Let us consider two major companies A and B competing for a business. The two companies A and B are regarded as players A and B respectively. The following matrix shows advertising strategies of both companies and their payoffs. Here each element of the matrix represents the gain of player A and the loss of Player B when the corresponding strategies are adopted by them which are represented by linguistic variables.

$$\text{Player B}$$

$$\text{Player A}\quad \begin{array}{c} \\ \textbf{Television} \\ \textbf{Radio} \\ \textbf{Press} \end{array} \begin{array}{c} \begin{array}{ccc} \textbf{Television} & \textbf{Radio} & \textbf{Press} \end{array} \\ \begin{pmatrix} \text{Very high} & \text{Very low} & \text{Extremely High} \\ \text{Extremely low} & \text{Low} & \text{Very low} \\ \text{High} & \text{Very High} & \text{Low} \end{pmatrix} \end{array}$$

Solution

The linguistic variables showing the qualitative data is converted into quantitative data using the following table. As the Payoff varies between 0 and 100, the minimum possible value is taken as 0 and the maximum possible is taken as 100.

Linguistic term	Value
Extremely low	(0,1,2,3,4,5,15)
Very low	(2,3,4,6,7,8,35)
Low	(4,5,7,9,10,13,43)
High	(5,7,10,12,14,16,74)
Very high	(6,7,8,12,14,16,53)
Extremely High	(10,12,18,20,25,30,65)

Step(1)

Replace the payoff matrix with linguistic variables by Heptagonal fuzzy numbers.

$$\text{Player B}$$

$$\text{Player A}\begin{pmatrix} (6,7,8,12,14,16,53) & (2,3,4,6,7,8,35) & (10,12,18,20,22,28,80) \\ (0,1,2,3,4,5,15) & (4,5,7,9,10,13,43) & (2,3,4,6,7,8,35) \\ (5,7,10,12,14,16,74) & (6,7,8,12,14,16,53) & (4,5,7,9,10,13,43) \end{pmatrix}$$

Step(2)

According to the definition of Heptagonal fuzzy number $\tilde{A}$, the measure of $\tilde{A}$ is calculated as

$$R(\tilde{A}_{hep}) = \frac{a_1+a_{2,}+a_3+a_4+a_5+a_6-a_7}{5}$$

Convert the given fuzzy problem into a crisp value problem by using the measure given by the definition.

Rank Values of Heptagonal Fuzzy Numbers in the fuzz game problem of Numerical Example

a_{11}= (6,7,8,12,14,16,53)	$R(hep_{11}) = \frac{6+7+8+12+14+16-53}{5} = 2$
a_{12}= (2,3,4,6,7,8,35)	$R(hep_{12}) = \frac{2+3+4+6+7+8-35}{5} = -1$
a_{13}= (10,12,18,20,22,28,80)	$R(hep_{13}) = \frac{10+12+18+20+22+28-80}{5} = 6$
a_{21}= (0,1,2,3,4,5,15)	$R(hep_{21}) = \frac{0+1+2+3+4+5-15}{5} = 0$
a_{22}= (4,5,7,9,10,13,43)	$R(hep_{22}) = \frac{4+5+7+9+10+13-43}{5} = 1$
a_{23}= (2,3,4,6,7,8,35)	$R(hep_{23}) = \frac{2+3+4+6+7+8-35}{5} = -1$
a_{31}= (5,7,10,12,14,16,74)	$R(hep_{31}) = \frac{5+7+10+12+14+16-74}{5} = -2$
a_{32}= (6,7,8,12,14,16,53)	$R(hep_{32}) = \frac{6+7+8+12+14+16-53}{5} = 2$
a_{33}= (4,5,7,9,10,13,43)	$R(hep_{33}) = \frac{4+5+7+9+10+13-43}{5} = 1$

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$$\text{The crisp game matrix is} \quad \text{Player A} \quad \begin{array}{c} \text{Player B} \\ \begin{pmatrix} 2 & -1 & 6 \\ 0 & 1 & -1 \\ -2 & 2 & 1 \end{pmatrix} \end{array}$$

Step(3) Check it saddle point

Maximum of row minima≠Minimum of column maxima

It has no saddle point.

In this matrix, We cannot apply matrix method and Dominance method. So, Using Pivot method to find out value of the game and optimum strategies.

Step (4) Pivot Method

(i) Adding 2 to each entry of the matrix is $\begin{pmatrix} 4 & 1 & 8 \\ 2 & 3 & 1 \\ 0 & 4 & 3 \end{pmatrix}$

(ii) We set up the tableau for the matrix as follows:

	B_1	B_2	B_3	
A_1	**4**	1	8	1
A_2	2	3	1	1
A_3	0	4	3	1
	-1	-1	-1	0

(iii) We must choose the pivot. Since all the columns have a negative number in the lower edge, we may choose any of these columns as the pivot column. Suppose we choose column 1. The pivot row must have a '+'ve number in this column, so it must be one of the top two rows. We compute the ratios of border numbers to pivot. For the first row it is ¼; for the second row it is ½. The former is smaller, so the pivot is in row 1. We pivot about 4 in the upper left corner.

(iv)The pivot itself gets replaced by its reciprocal, namely ¼. The rest of the numbers in the pivot row are simply divided by the pivot, giving ¼,2and ¼. Then the rest of the numbers in the pivot column are divided by the pivot and changed in sign. The remaining q numbers are modified by subtracting (r.c)/p for corresponding r and c.

	A_1	B_2	B_3	
B_1	1/4	1/4	2	1/4
A_2	-1/2	**5/2**	-3	1/2
A_3	0	4	3	1
	1/4	-3/4	1	1/4

(v) We interchange the labels of the pivot row and column. Here we interchange A_1 and B_1.

(vi) We check for '-' ve entries in the lower edge. Since there is one, we return to step (3).

(vii)We choose next pivot in column 2, since it has the unique negative number in the lower edge. All 3 numbers in this column are '+"ve. We find the ratios of border numbers to pivot for rows 1,2,3 to be 1,1/5 and ¼. The smallest occurs in the second row, so we pivot about the 5/2 in the second row,second column. Repeat step (4) and (5), we obtain

	A_1	A_2	B_3	
B_1	0.3	-0.1	2.3	0.2
B_2	-0.2	**0.4**	-1.2	0.2
A_3	0.8	-1.6	7.8	0.2
	0.1	0.3	0.1	0.4

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All values on the lower edge are non-negative.

(viii) The value of the game is the reciprocal of 0.4= 5/2

Since A_3 is on the left on the final tableau, the optimal p_3 is zero.The optimal p_1 and p_2 are the ratios, $\frac{0.1}{0.4}=\frac{1}{4}$ and $\frac{0.3}{0.4}=\frac{3}{4}$

Therefore, The optimal mixed strategy for player A is (p_1,p_2,p_3)= (1/4,2/4,0)

Since B_3 is on the top in the final tableau, the optimal q_3 is zero. The optimal $q_1$and q_2 are the ratios, $\frac{0.2}{0.4}=\frac{1}{2}$ and $\frac{0.2}{0.4}=\frac{1}{2}$

Therefore, The optimal mixed strategy for player B **is (q_1,q_2,q_3)= (1/2,1/2,0)**

Step (5)

Therefore, the game with original matrix has the same optimal mixed strategies but the value of the game= (5/2)-2 =1/2.

CONCLUSION

In this paper a fuzzy game problem with linguistic variables as payoffs is considered. These linguistic variables are replaced by Heptagonal fuzzy number and using ranking to these payoffs, the fuzzy game problem is transformed into a crisp one. The crisp game problem can be solved by any traditional method. Here it is solved by Pivot method.

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